APPLECROSS
SENIOR HIGH SCHOOL

YEAR 11 SPECIALIST MATHEMATICS

TEST 1 – COMBINATORICS

Wednesday, February 26, 2020

Name: Solutions

Structure of this paper:

Number of Questions	Total Marks	Working Time	Your Score	%
9	49	50 min		

Course Average: _____ %

Question 1: **(4 marks)**

a) How many subsets does the set {a, b, c, d, e, f, g, h} contain? (1 mark)

$2^8 = 256$ ✓

(b) How many subsets of {a, b, c, d, e, f, g, h} have 3 elements? (1 mark)

$\binom{8}{3} = 56$ ✓

(c) Ruby has one 5 cent, one 10 cent, one 20 cent and one 50 cent piece. How many different sums of money can Ruby make if she uses at least one coin? (2 marks)

✓

$2^4 - 1 = 16 - 1$

$= 15$ ✓

Question 2: **(3 marks)**

a) How many students must be taken at random from a class to be sure of getting two with birthdays in the **same month?** (1 mark)

13 ✓

b) In the first 100 words of a novel, at least how many of the words will start with the same letter? (2 marks)

$\frac{100}{26}$ ✓ $= 3.846\ldots$

$\therefore$ at least 4 words ✓

Question 3: **(6 marks)**

A hand of five cards is dealt from a deck of 52 playing cards. How many hands contain exactly:

a) two clubs (1 mark)

$\binom{13}{2}\binom{39}{3} = 712842$ ✓

b) three spades (1 mark)

$\binom{13}{3}\binom{39}{2} = 211926$ ✓

c) two clubs and three spades (1 mark)

$\binom{13}{2}\binom{13}{3} = 22308$ ✓

d) two clubs or three spades? (3 marks)

$712842 + 211926 - 22308$ ✓ ✓

$= 902460$ ✓

Question 4: **(5 marks)**

A pizza restaurant offers the following toppings: onion, capsicum, mushroom, olives, ham and salami.

a) How many different kinds of pizza can be ordered with

(i) three different toppings? (1 mark)

$\binom{6}{3} = 20$ ✓

(ii) three different toppings including ham? (1 mark)

$\binom{1}{1}\binom{5}{2} = 10$ ✓

(iii) any number of toppings between none and all six? (1 mark)

$2^6 = 64$ ✓

b) Another pizza restaurant boasts that they can make more than 200 varieties of pizza. What is the smallest number of toppings they could use? (2 marks)

$2^n = 200$ ✓

$n = 8$ ✓

Question 5: **(7 marks)**

a) How many seven letter arrangements are there of the letters of the word QUALITY if in each arrangement each letter must be used just once? (1 mark)

$7! = 5040$ ✓

b) How many of these arrangements

i) have the Q and the U next to each other, (1 mark)

$6!2! = 1440$ ✓

ii) have the Q and the U separated, (1 mark)

$7! - 6!2! = 3600$ ✓

iii) end with Y and have Q and U next to each other, (1 mark)

$5!2!1! = 240$ ✓

iv) start with Q or end with Y? (3 marks)

Start Q = $6!$ }
End Y = $6!$ } ✓
Q ... Y = $5!$

$\therefore \quad 6! + 6! - 5!$ ✓
$= 1320$ ✓

Question 6: **(6 marks)**

a) How many ways can the thirteen letters of the word PARALLELOGRAM be arranged in a row?

$$\frac{13!}{3!3!2!} = 86486400$$ ✓ ✓

b) How many four letter permutations are there of the letters of the word RECTANGLE?

RECTANGL
E

all different $8 \times 7 \times 6 \times 5 = 1680$ ✓

EE__
E_E_
E__E
_EE_
_E_E
__EE

✓ $6 \times 7 \times 6 \times 1$ (E's) $= 252$ ✓

Total Number: 1932 ✓

Question 7: **(5 marks)**

A dance team of 11 students is to be chosen from a squad of 15 students
How many different teams can be chosen that include the following?

a) Cindy and Jennifer, (1 marks)

$\binom{2}{2}\binom{13}{9} = 715$ ✓

b) Cindy or Jennifer (4 marks)

C J

$\binom{1}{1}\binom{1}{0}\binom{13}{10}$ ✓

C J

$\binom{1}{0}\binom{1}{1}\binom{13}{10}$ ✓

C J

$\binom{1}{1}\binom{1}{1}\binom{13}{9}$ ✓

$\therefore \binom{13}{10} \times 2 - \binom{13}{9}$

$= 1287$ ✓

Question 8: **(9 marks)**

a) 12321 is a palindromic number because it reads the same backwards as forwards. How many palindromic numbers have five digits? (2 marks)

$9 \times 10 \times 10 \times 1 \times 1 = 900$ ✓✓

b) How many ways can six boys and six girls be arranged in a row if:

(i) boys and girls sit in alternate positions? (2 marks)

BGBG....

$6!6! \times 2!$ ✓

$= 1036800$ ✓

(ii) boys sit together and girls sit together? (2 marks)

BB..BGG..G $\times 2!$

$6!6! \times 2!$ ✓

$= 1036800$ ✓

c) Six people are to be seated in a row. Calculate the number of ways this can be done so that two people, A and B, always have exactly one person between them?

(3 marks)

A.4B.3.2.1
4A3B21
43A2B1
432A1B

(A,B or B,A)

$4! \times 4 \times 2! = 192.$

Question 9 **(4 marks)**

A class has 50 students.

(a) How many students need to be chosen to ensure that there are:

(i) two students who are born on the same day of the week?

8

(ii) five students who are born on the same day of the week?

29

(b) There are at least x students who are born on the same day of the week. Find x. Justify your answer.

$50 = 7 \times 7 + 1$

That remainder means that at least 8 students born on the same day.

Combinatorics

Combinations

Number of arrangements: (of n different objects in an ordered list)

$$n(n-1)(n-2)\times.....\times 3\times 2\times 1 = n!$$

Number of combinations: (of r objects taken from a set of n distinct objects)

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}; \qquad \binom{n}{r} = \binom{n}{n-r}; \qquad \binom{n}{0} = 1$$

Number of permutations: (of r objects taken from a set of n distinct objects)

$${}^{n}P_{r} = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Number of permutations with some identical objects: $\dfrac{n!}{r_1!r_2!r_3!...}$

Inclusion – exclusion principle: $|A\cup B| = |A| + |B| - |A\cap B|$

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$